# Kinetic equilibration in heavy ion collisions: the role of elastic processes

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We discuss the question of thermalization during the very early stages of a high energy heavy ion collision. We review a recent study where we explicitely showed that, contrarily to a widely used assumption, elastic collisions between the produced partons are not sufficient to rapidly drive the system toward local kinetic equilibrium. We then briefly discuss recent developments concerning the description of kinetic equilibration and comment on some open issues related to phenomenology.

## 1. Introduction

Present and upcoming high energy heavy ion collisions at RHIC and LHC offer a great opportunity to test our understanding of the dynamics of strong interactions in unusually extreme conditions. A central question is that of the possible formation of a locally equilibrated state of deconfined matter, the so-called quark-gluon plasma (QGP). By now, there is no doubt that during the very early stages of such collisions the relevant degrees of freedom are quarks and gluons and it is rather clear that the initially produced partons undergo many subsequent rescatterings [1]. It is a question of great interest for interpreting experimental data to know whether this dense gas of partons thermalizes before hadronization and, if yes, on which time scale.

It is usually assumed that elastic scatterings between the produced partons, which randomize their momenta, rapidly drive the system toward a state of local kinetic equilibrium, on time scales $\lesssim 1$ fm [2,3]. Under this assumption, the subsequent evolution toward local thermodynamic equilibrium consists in the equilibration of the chemical composition of the gas through inelastic number changing processes [2]. However, the estimate of the kinetic equilibration time scale on which the previous picture relies is not satisfying. First, it is based on the assumption of a gluonic system near equilibrium [3], whereas in an actual collision the produced gluons are far away from equilibrium. Second, it neglects the effect of longitudinal expansion at early times, which may have dramatic consequences: on the one hand, longitudinal free-streaming tends to destroy the isotropy of the momentum distribution; on the other hand, expansion dilutes the system and the collisions between partons, which drive the system toward equilibrium, become more and more rare as time goes on. If the expansion is too strong, it may even be that the system never reaches equilibrium [4]. Moreover, the importance of the initial condition which characterizes the partonic system just after the collision has been emphasized [4,5]. Recent developments

*This talk is based on a work done in collaboration with D. Schiff [9] concerning a realistic description of the initial state have triggered more reliable studies of the microscopic evolution of the gluon gas [5–11]. Here we report on a detailed study where we explicitely showed that the assumption of a rapid kinetic equilibration due to elastic collisions is not correct. We summarize the main hypothesis and results of this work; more details can be found in [9]. We then discuss the question of thermalization at the light of recent works.

## 2. Kinetic equilibration: the role of elastic processes

The matter produced in the central rapidity region is essentially made of gluons with transverse momentum $p_t \sim 1-2$ GeV [5,12]. We compare the case where initial conditions are given by the saturation scenario, where the non-abelian Weizsäcker-Williams fields of the incoming nuclei materialize in on-shell gluons after the collision [5], to the case where the initial gluons are produced incoherently in semi-hard (perturbative) processes, the so-called minijet scenario [12].

Following Mueller, we assume that already at early times a Boltzmann equation can be written for the partonic phase space distribution. Including only $2 \to 2$ elastic processes in the small-scattering angle limit, one obtains, in the leading logarithmic approximation, the QCD analog of the Landau collision integral [5]. We use a relaxation time approximation, which allows us to make further analytical progress. In particular, we consider simple observables which directly probe kinetic equilibration and for which all the phase-space integrals can be computed analytically. One is then left with a set of coupled one-dimensional dynamical equations, which are easily solved numerically. We compute the time-dependent relaxation time in a self-consistent way and we carefully take care of conservation laws. Comparing with the results of [6], where the exact solution is worked out numerically in the saturation scenario, one can assess the reliability of our approach, which appears to be quite good. We then apply this method to the minijet scenario [12,8]. We argue that, to characterize kinetic equilibration, a reliable criterium is to test the isotropy of various observables. As a consequence we find in particular that for both initial conditions the system does not reach equilibrium at RHIC energies. We test the robustness of our results by studying their sensitivity to the details of our description. We argue in particular that, due to the fragility of the weak coupling approximation, it appears difficult to obtain definite conclusions at LHC energies. In any case, we obtain reliable lower limits for the equilibration time. Requiring that the ratio of longitudinal and transverse pressures – which should be 1 in equilibrium – be greater than 0.8, we get:

- *Saturation scenario:* $t_{\rm elastic} \gtrsim 6$ fm at RHIC and $t_{\rm elastic} \gtrsim 3$ fm at LHC;
- *Minijet scenario:* $t_{\rm elastic} \gtrsim 10$ fm at RHIC and $t_{\rm elastic} \gtrsim 5$ fm at LHC.

For comparison, recall that, for example at RHIC, the time during which a partonic description makes sense is at most of the order of 10 fm.

It is interesting to quantify our results with respect to the analytic estimate of the equilibration time obtained by Mueller in the saturation scenario [5]: $t_{\rm elastic} \simeq c\, Q_S^{-1} \exp\sqrt{1/\alpha_S}$, where $\alpha_S$ is the strong coupling constant, $Q_S$ is the saturation scale, and $c$ is a numerical constant of order one. Using $Q_S$ as the scale fixing the value of the running coupling constant and inserting realistic values ($Q_S \simeq 1$ GeV at RHIC and $Q_S \simeq 2$ GeV at LHC) one gets $t_{\rm elastic} \simeq c$ fm at RHIC and $t_{\rm elastic} \simeq 0.6\, c$ fm at LHC. Comparing to our results, one concludes that $c \gtrsim 5$. This number, which is in principle of little importance in the weak coupling limit, plays a crucial role for values of the parameters appropriate for RHIC and LHC energies.

To conclude, we have shown that the kinetic equilibration time due to elastic processes only is at least an order of magnitude bigger than the typical 1 fm estimate usually assumed. Of course, this does not mean that kinetic equilibrium is not achieved in a real collision, but this implies that the standard picture of the space-time evolution of the system, described in the introduction, has to be modified. Important progress are being made in this respect and a more reliable picture is already emerging. In the remaining sections, we try to summarize and discuss the current state of this rapidly evolving subject.

## 3. Beyond elastic processes: the bottom-up scenario

In the context of the saturation picture of gluon production, the authors of [10] have pointed out the crucial role played by the dominant $2 \to 3$ branching process and have developed a complete description of kinetic equilibration: the so-called bottom-up scenario, which predicts that the equilibration time is parametrically given by: $t_{\rm eq} \sim Q_S^{-1}\,(1/\alpha_S)^{13/5}$. Inserting realistic values as before, on obtains typically: $t_{\rm eq} \sim 2$ fm both at RHIC and LHC. Moreover, from a comparison with the observed multiplicities at RHIC together with reasonable assumptions, the same authors infer the values: $t_{\rm eq} \simeq 3.2{-}3.6$ fm [10]. Therefore, we see that, once $2 \to 3$ processes are taken into account, kinetic equilibrium is likely to be achieved already at RHIC, however on a time scale which is still non negligible. This implies in particular that theoretical calculations addressing the various experimental signatures of deconfined matter should take into account the out of equilibrium evolution of the system [13]

To complete the picture, the important next step is to include quark-antiquark pair production and to describe chemical equilibration. Previous studies indicate that the corresponding equilibration time is not small [2], but this has still to be investigated in a bottom-up like picture.

## 4. Evidence for early thermalization in heavy ion collisions?

Finally let us discuss a phenomenological puzzle, recently reported in the literature [14]: experimental data on hadron spectra and elliptic flow appear to be very well described by perfect hydrodynamics, if one assumes that local thermal equilibrium is achieved already at a time 0.6 fm after the collision. As we have seen above, such a short time scale is however difficult to understand on the basis of perturbative QCD, which should be at least qualitatively reliable here.[2] Moreover, as pointed out by Mueller [1], this is at the limit of applicability of an hydrodynamic description, this time being only slightly bigger than the time it takes for a typical gluon of this plasma to be produced. Nevertheless, the impressive agreement with data reported in [14] is striking and needs to be understood.

One can follow two different directions: the first one is to try to understand how such a fast thermalization could occur. A recent suggestion is that of non-perturbative

[2]The initial temperature required to match the data in [14] is $T \simeq 330$ MeV, which correspond to partons having a typical momentum $p \sim 3T \simeq 1$ GeV.

processes leading to very rapid production of large amount of entropy [15]. Also, non-equilibrium phenomena may lead to so-called prethermalization, where some degrees of freedom rapidly exhibit a thermal behavior while the system as a whole is still out-of-equilibrium [16,17]. Finally, another interesting idea is that the partons may already be "thermalized" in the incoming nuclear wave-function [18]. The second direction is to investigate to which extent this agreement relies on the assumption of a completely equilibrated system. An interesting study of this type has been made in [19], where the authors considered the opposite limit, namely that of a system which is only transversally thermalized and which undergoes free-streaming in the longitudinal direction. Their results deviate from the ones obtained with perfect hydrodynamics. However, it is not clear how fast the results corresponding to intermediate situations would go from one limit to the other. In this respect, it would be very interesting to consider more realistic situations, where the system develops more and more longitudinal pressure as time goes on (see *e.g.* [9]).

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